1. Find the exact value of the expression or say why it is undefined.
   (a) \( \sin \left( \frac{5\pi}{6} \right) = \frac{1}{2} \)  
   (b) \( \cos \left( \frac{-\pi}{2} \right) = 0 \)  
   (c) \( \tan \left( \frac{\pi}{3} \right) = \sqrt{3} \)  
   (d) \( \sin \left( -\frac{3\pi}{4} \right) = -\frac{\sqrt{2}}{2} \)  
   (e) \( \sec(0) = 1 \)  
   (f) \( \tan \left( \frac{2\pi}{3} \right) = -\sqrt{3} \)  
   (g) \( \cos \left( \frac{4\pi}{3} \right) = -\frac{1}{2} \)  
   (h) \( \csc \left( -\frac{\pi}{4} \right) = -\sqrt{2} \)  

2. Find the exact value of the expression or say why it is undefined.
   (a) \( \sin(x^2\pi), \ x = \frac{1}{2} \)  
      \( \Rightarrow \sin \left( \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2} \)  
   (b) \( u \cos^2 u, \ u = \frac{\pi}{6} \)  
      \( \Rightarrow \frac{\pi}{6} \cos^2 \frac{\pi}{6} = \frac{\pi}{6} \left( \frac{\sqrt{3}}{2} \right)^2 = \frac{\pi}{8} \)  
   (c) \( \cot(\pi \sin a), \ a = \frac{\pi}{2} \)  
      \( \Rightarrow \cot \left( \frac{\pi}{2} \right) = \cot(\pi) = \frac{\cos \pi}{\sin \pi} = \text{undefined, because } \sin \pi = 0. \)  
   (d) \( \frac{\cos^2 x + \sin^2 x}{x}, \ x = 20 \)  
      \( \Rightarrow \frac{1}{x} = \frac{1}{20} \)  

3. Find all solutions of the given equation on the indicated interval.
   (a) \( 2 \cos x = 1, \ 0 \leq x \leq 2\pi \)  
      \( \Rightarrow \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3} \) or \( x = \frac{5\pi}{3} \)  
   (b) \( \sin(3\theta^2) = \frac{\pi}{2}, \ 0 \leq \theta \leq 2\pi \)  
      \( \Rightarrow \sin x = 1 \)  
      \( \Rightarrow \text{no solutions, since } \sin x \leq 1 \) for all \( x. \)  
   (c) \( u \cos \left( \frac{u}{4} \right) + u = 0, \ -4\pi \leq u \leq -2\pi \)  
      \( \Rightarrow u \left( \cos \left( \frac{u}{4} \right) \right) + 1 = 0 \Rightarrow u = 0 \) or \( \cos \left( \frac{u}{4} \right) = -1. \)  
      \( \Rightarrow \sin^2 t = \frac{1}{4} \Rightarrow \sin t = \pm \frac{1}{2}, \) so \( t = -\frac{\pi}{6} \) or \( t = \frac{\pi}{6}. \)  
      \( \Rightarrow u = 4 \) or \( u = -4 \pi. \)  

4. Simplify the given expression.
   (a) \( \sin x \tan x + \cos^2 x \sec x \)  
      \( = \sin x \left( \frac{\sin x}{\cos x} \right) + \cos^2 x \left( \frac{1}{\cos x} \right) \)  
      \( = \frac{\sin^2 x + \cos^2 x}{\cos x} \)  
      \( = \frac{1}{\cos x} = \sec x \)  
   (b) \( \frac{\sin 2x}{\sin x} - \cot x \sin x \)  
      \( = \frac{2 \sin x \cos x}{\sin x} - \left( \frac{\cos x}{\sin x} \right) \sin x = 2 \cos x - \cos x \)  
      \( = \cos x. \)  

5. (a) Suppose that \( \sin \theta = 0.6 \) and \( \frac{\pi}{2} < \theta < \pi. \) Find \( \cos \theta, \tan \theta, \) and \( \sec \theta. \)  
      Since \( \sin^2 \theta + \cos^2 \theta = 1, \ \cos^2 \theta = 1 - 0.6^2 = 0.64. \)  
      Therefore, \( \cos \theta = \pm 0.8. \) Since cosine is negative for \( \frac{\pi}{2} < \theta < \pi, \ \cos \theta = -0.8. \)  
      Then \( \tan \theta = \frac{0.6}{-0.8} = -0.75 \) and \( \sec \theta = \frac{1}{-0.8} = -1.25. \)  

(b) Suppose that \( \tan x = -\sqrt{3} \) and \( \frac{3\pi}{2} < x < 2\pi. \) Find \( \sin x, \cos x, \) and \( \csc x. \)  
      Since \( \tan x = -\sqrt{3} = -\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sin x}{\cos x}, \) then \( \sin x = -\frac{\sqrt{3}}{2} \) and \( \cos x = \frac{1}{2}. \) (the signs are opposite.)  
      Since cosine is positive for \( \frac{3\pi}{2} < x < 2\pi, \) it must be that \( \sin x = -\frac{\sqrt{3}}{2}, \ \cos x = \frac{1}{2}. \)  
      and \( \csc x = \frac{1}{\sin x} = -\frac{2}{\sqrt{3}}. \)
Notes on the Unit Circle Diagram

* The circle has radius 1 and points \((x, y)\) on the circle satisfy the equation \(x^2 + y^2 = 1\).
* Angles are displayed inside the circle (in degrees and radians). They are measured counter-clockwise starting from the positive x-axis (“upwards”). Negative angles (not displayed) are measured clockwise starting from the positive x-axis (“downwards”).
* For any point \((x, y)\) on the circle: \(x = \cos \theta\) and \(y = \sin \theta\) (displayed outside the circle). For example,
  \[-\frac{1}{2} = \cos \left(\frac{2\pi}{3}\right), \quad \frac{\sqrt{3}}{2} = \sin \left(\frac{2\pi}{3}\right)\].

* The most important quadrant is in the upper right of the circle (1st quadrant of the x-y plane). If you know the information given in this part of the circle (angles and their sine and cosine values) then by symmetry you can determine the information given for the rest of the circle.
6. Match each formula to one of the given graphs, below.

(a) \( y = \sin x + 1 \)  \hspace{1cm}  (b) \( y = \sin 2x \)  \hspace{1cm}  (c) \( y = 2\sin 2x \)  \hspace{1cm}  (d) \( y = \sin \left( \frac{x}{2} \right) \)

(e) \( y = \cos 2x \)  \hspace{1cm}  (f) \( y = 2\cos x \)  \hspace{1cm}  (g) \( y = 2\cos \left( \frac{x}{2} \right) \)  \hspace{1cm}  (h) \( y = \frac{\cos x - 1}{2} \)

(I) \( b \) \hspace{1cm} (II) \( e \)  \hspace{1cm} (III) \( g \)  \hspace{1cm} (IV) \( f \)

(V) \( a \)  \hspace{1cm} (VI) \( c \)  \hspace{1cm} (VII) \( d \)  \hspace{1cm} (VIII) \( h \)
7. Let \( f(x) = \cos x \), \( g(x) = x^2 + 2 \), and \( h(x) = \frac{1}{\sqrt{x}} \). Find and simplify the given expression.

(a) \( f(g(x)) \)  
(b) \( g(f(x)) \)  
(c) \( h(2 - f^2(x)) \)  
(d) \( g(h(f(x))) \)  
(e) \( f(g(h(x))) \)

(a) \( f(g(x)) = \cos(x^2 + 2) \)

(b) \( g(f(x)) = \cos^2 x + 2 \)

(c) \( h(2 - f^2(x)) = h(2 - \cos^2 x) = \frac{1}{\sqrt{2 - \cos^2 x}} \)

(d) \( g(h(f(x))) = g\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\cos x} + 2 \)

(e) \( f(g(h(x))) = f\left(\frac{1}{x^2} + 2\right) = \cos\left(\frac{1}{x^2} + 2\right) \)

8. A ladder 10 feet long leans against a vertical wall. Let \( \theta \) be the angle between the top of the ladder and the wall, and let \( x \) be the distance from the bottom of the ladder to the wall. (Assume that \( x > 0 \).)

(a) Make a diagram that incorporates the given information.
(b) Express \( \sin \theta \) in terms of \( x \).
(c) If the ladder slides down the wall, how does \( x \) change: does it increase, decrease, or stay the same? How does \( \sin \theta \) change? How does \( \theta \) change?
(d) Express \( \cos \theta \) in terms of \( x \).
(e) If the ladder slides down the wall, how does \( \cos \theta \) change?

(a) \[
\begin{align*}
\theta & \quad \text{10} \\
\ldots & \quad \ldots \\
x & \quad \ldots \\
\end{align*}
\]

(b) In a right triangle, \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \). For the triangle in the diagram we thus have \( \sin \theta = \frac{x}{10} \).

(c) If the ladder slides down the wall (see figures) \( x \) increases, so \( \sin \theta = \frac{x}{10} \) increases and \( \theta \) increases.

(d) Using the identity \( \cos^2 \theta + \sin^2 \theta = 1 \), \( \cos^2 \theta = 1 - \frac{x^2}{100} \), so \( \cos \theta = \sqrt{1 - \frac{x^2}{100}} = \frac{\sqrt{100 - x^2}}{10} \).

(Alternate method: let \( y \) be the side of the triangle along the wall. Then \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{y}{10} \) Now use the Pythagorean Theorem.)

(e) If the ladder slides down the wall, then \( \theta \) increases which means \( \cos \theta \) decreases. This is because \( \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{y}{10} \), and as \( \theta \) increases, \( y \) decreases.