Measuring Direct Discrimination in Labor Markets Using a Frontier Approach: Evidence from CPS Female Earnings Data*

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I. Introduction

Numerous studies of earnings differentials have attempted to measure discrimination against females (or minorities) by decomposing the total differential between males and females (or whites and non-whites) into an explained and unexplained portion. Typically, these studies follow the work of Oaxaca [8]. However a number of criticisms can be raised with these techniques [9; 5; 4] of which two are particularly important: that the reduced form wage equations for the two groups will not be identical in every case and that there may be large and unobservable differences between the characteristics of the two groups for which the wage equations cannot account. Either of these problems are sufficient to make it impossible to use residual analysis to measure discrimination. Attempts to address these problems and obtain reliable and consistent estimates of the extent of discrimination are important for policy purposes.

Low and Villegas [7] address the first of these problems by suggesting the use of hedonic wage equations which contain only human capital characteristics (or proxies) and characteristics which directly affect the hedonic parameters. In this framework perfect substitution between the workers is assumed. Their work however, leaves the second problem unresolved.

In this paper a stochastic earnings frontier[1] with a discrimination discount is used to estimate discrimination against women. An earnings frontier for females that depends upon marginal productivity (as measured by human capital and labor market characteristics) of females is estimated. Discrimination is assumed to be the amount female earnings are removed from the frontier, less any labor market inefficiency. Since these estimates will depend solely on female data they do not depend upon an assumption of similarity between males and females. This procedure allows

*The authors wish to thank James Rebitzer for making the data available for this paper and Solomon Polachek, Betty Collier Watson, the participants in the Eastern Economic Association Meetings Session on "Economics of Gender Wage Differentials" in Boston, March 1988, and an anonymous referee for their helpful comments.

1. Boehm and Hofler [3] use a frontier approach to estimate racial discrimination in housing, while Robinson [12] applies this technique to sex discrimination across regions.

estimates of discrimination to be obtained without assuming away all differences between the favored and unfavored groups. Initial estimates using this technique suggests that discrimination lowers average hourly earnings of women[2] by about twenty-six percent.

II. Measuring Discrimination through Unexplained Differentials

Two basic types of discrimination are at issue: (1) the extent to which males and females (or whites and non-whites) that have identical characteristics are treated differently in terms of labor market outcomes, and (2) the extent to which the indirect discrimination of socialization and acculturation of males and females leads to economic decisions that leave males and females with different levels of human capital, work attachment, occupational aspirations, or motivation. It is the intent of this paper to measure only type (1). Traditional residual analysis, as described below, while attempting to measure (1), may in fact measure some combination of (1) and (2).

To measure the level of unexplained differential between males and females using either the traditional or hedonic approach requires the estimation of a wage equation for males and females. The total differential between male and female earnings can be decomposed into those differentials due to differences in the characteristics of males and females and differences due to different returns to human capital (provided, of course, the coefficient estimates measure return to human capital and not other factors). In order to complete this decomposition estimates of female earnings in the absence of discrimination must be computed by applying the male wage equation parameters to mean female characteristics or vice versa.

This procedure assumes that the male parameters are the correct parameters to apply to the female characteristics in the absence of discrimination. This assumption may be incorrect if there are unmeasurable differences between males and females not contained in the data. For example, if females have lower labor force attachment then males and consequently lower work effort, one would not expect that males and females with the same number of years of education would receive the same hourly earnings because they would not have the same amount of human capital. Goldin and Polachek [5] provide a thorough discussion of this point. In this case using the male parameters to predict female earnings would over-estimate the level of female earnings in the absence of discrimination. Of course the discrimination we mean is direct discrimination. The difference in labor force attachment may be the result of indirect discrimination. Residual analysis generates (both in the hedonic and reduced form cases) a measure of discrimination under the assumption males and females are identical except with respect to measured characteristics differences.

Clearly, an approach to measuring direct discrimination that eliminates the need to make these assumptions about males and females (or whites and non-whites) would represent an improvement over alternative methodologies. Robinson [12] and Boehm and Hofler [3] suggest such an approach. Stochastic frontier estimation can be used to estimate the extent to which earnings of one group, females, are removed from a stochastic earnings frontier that is determined by marginal productivity.[3]

2. Non-union, full-time, private sector women.

3. This approach has also been used by Boehm and Hofler [2] and by Polachek and Yoon [10] to measure labor market inefficiency.

III. Methodology and Data

The reduced form or hedonic wage equation for females relates earnings to human capital characteristics or marginal productivity (or in the case of the reduced form equation to market supply and demand as well). This relationship is shown in equation (1).

$$\text{Log}(EARNINGS_{female}) = f(HUMAN\ CAPITAL) + e\,. \tag{1}$$

In equation (1) earnings are related to acquired, relevant human capital plus a stochastic error term. Direct discrimination against females can be defined as the extent (on average) that female earnings lie below the earnings frontier defined by marginal productivity and represented in equation (1). To obtain an equation that can be estimated to determine the amount that female earnings lie beneath the frontier, equation (1) can be modified by the addition of an additional error term that is always negative and is distributed as truncated half-normal.

$$\text{Log}(EARNINGS_{female}) = f(HUMAN\ CAPITAL) + e_u + e_v, \tag{2}$$

where e_u is the normal error and e_v is the truncated error. All females have earnings that lie at or below the frontier.

In order to estimate direct discrimination one does not need to assume identical unmeasured characteristics of males and females. However, in order to attribute the entire disturbance to discrimination it must be assumed that the wage setting process is not asymmetrically inefficient [10]. Clearly, any symmetric inefficiency would be captured in the non-truncated stochastic term. If the inefficiency were asymmetric, this would mean that female wages could lie below the female frontier even without discrimination. If this is the case the expected value of the half-normal error term measures the total of expected discrimination and expected inefficiency. Obviously, if the expected value of the error term is not significantly different from zero, there is no evidence of discrimination (or asymmetric inefficiency). On the other hand, a significant negative expected value would be an estimate of the maximum direct discrimination that could be present.[4] Clearly any attempt to estimate such a model will be conditional on the assumptions about the distribution of the discrimination error term. Consideration of the results should be with this fact in mind.

This definition of discrimination does not necessarily imply a particular model of discrimination, rather a means by which discrimination can be measured. Measuring discrimination in this fashion has a number of appealing implications:

1. One need not impose the restriction that males and females are identical in all unmeasured qualities. (Although the assumption that there are no asymmetric inefficiencies in wage determination must be maintained to attribute the entire variance to direct discrimination.)

2. Discrimination is applied differently to different females. Some individuals will lie far below the frontier, while others will be much closer. This view of discrimination seems more consistent with reality than modeling sex differences in earnings with a dummy variable, or differing returns to human capital, both of which assume that all females are treated identically and that discrimination is applied in a systematic fashion to all individuals.

4. Polachek and Yoon [10] present a labor market inefficiency model. They estimate a two-tiered earnings frontier with two truncated error terms: one measuring employer ignorance about reservation wages and a second measuring employee ignorance of wage offers. This model might be viewed as a competing model of asymmetric wage determination. However, if we assume that the distribution of wage offers is a function of different levels of discrimination between employers and that employee ignorance is a necessary condition for discrimination to persist, than the two models in fact measure the same variance on the employee side.

3. Goldin and Polachek [5] point out that the increasing heterogeneity of the female work force over time has led to an increase of the residual as a percentage of the male-female wage gap. Our procedure is not sensitive to the total size of the residual, but rather to its skewness. Thus estimates of discrimination obtained in this manner should be more consistent over time. The problem of the heterogeneity of female workers is compounded by the homogeneous treatment of human capital variables such as tenure and education. It seems quite likely that the returns to tenure will be quite different across occupational groups. In order to control for this interaction terms will be introduced to allow the earnings impact of tenure to vary across occupations.

4. Robinson [12] derived a model of this sort from the statistical discrimination literature. Aigner and Cain [1] develop a form of the statistical discrimination model after rejecting the simpler Phelps [11] model. The model is adapted by adding the notion of risk aversion into the decisions of employers. Employers not only set wages equal to expected ability, they also set wages so as to minimize the damage mistakes might make to the firm. In this case employers take risk (as measured by the variance of the true ability of the group to which the individual belongs) into account. In this version of the statistical discrimination model risk adverse employers offer wages of expected ability less a risk discount. The earnings equation with a risk discount is presented in (3):

$$\text{Log}(EARNINGS_{female}) = f(HUMAN\ CAPITAL) + e_u - R \tag{3}$$

where R is the risk discount applied to females. If it is assumed that R is a complex function of employer attributes and therefore that it varies over employers, then R can be replaced in equation (3) by e_v, a truncated error term. The resulting wage equation is given by equation (2).

Equation (2) can be estimated to obtain parameter estimates for the coefficients on the human capital terms and the variances of the normal and half-normal error terms. The amount of wage discrimination females suffer on average can be measured by determining the expected value of the half-normal error term. Equation (2) can be estimated using a maximum-likelihood technique. The log-likelihood function for (2) was derived by Aigner, Lovell, and Schmidt [2] and is shown below in equation (4):

$$\begin{aligned}\log \mathbf{L}(\mathbf{y}|\boldsymbol{\beta}, s_u^2, s_v^2) = {} & N \ln(2^{1/2}/\pi^{1/2}) + N \ln(1/(s_u^2 + s_v^2)^{1/2}) \\ & + \sum_{i=1}^{N} \ln[1 - F^*(e_i(s_v^2/s_u^2)^{1/2}/(s_u^2 + s_v^2)^{1/2})] \\ & - 1/(2(s_u^2 + s_v^2)) \sum_{i=1}^{N} e_i \end{aligned} \tag{4}$$

where s_v is the standard deviation of the truncated error term, s_u is the standard deviation of the normal error term, e_i is the residual of the ith individual, and F^* is the cumulative normal distribution.[5]

In order to estimate (2) data from the May 1983 CPS was used and the TSP maximum likelihood estimation routine was employed. The sample consisted of full-time,[6] non-union[7], females in the private non-agricultural work force in May of 1983. The dependent variable was log of hourly earnings. Three specifications were employed:

5. Clearly the results of this estimation are conditional on our assumptions about the distribution of the discrimination error term.

6. Where a worker is full-time if she works 35 or more hours per week.

7. Union workers were excluded for two reasons: discrimination in union and non-union jobs may be quite different and estimating union/non-union differentials in a simplistic fashion might bias other estimates.

Table I. Maximum Likelihood Estimates of the Earnings Frontier for Females, Dependent Variable Log of Hourly Earnings

	Hedonic Model		Reduced Form (1)		Reduced Form (2)	
Variable	Estimate	T-Value	Estimate	T-Value	Estimate	T-Value
Intercept	0.139	2.30	0.368	5.87	0.381	5.97
Age	0.039	14.09	0.037	13.86	0.037	13.76
Age^2/100	−0.048	−14.10	−0.045	−13.83	−0.045	−13.75
Tenure	0.037	19.28	0.033	17.85	—	—
Tenure^2/100	−0.076	−12.54	−0.070	−12.07	—	—
Education	0.077	36.44	0.064	26.61	0.063	26.36
Race	0.065	3.89	0.072	4.36		
Plant Size	—	—	0.205	14.25	0.205	14.24
Services (1)	—	—	−0.105	−7.80	−0.126	−6.66
Executive (2)	—	—	0.127	6.99	0.127	4.56
Operatives (3)	—	—	−0.107	−5.57	−0.089	3.02
Technical (4)						
Tenure1	—	—	—	—	0.037	12.22
Tenure^{2}1	—	—	—	—	−0.079	−6.88
Tenure2	—	—	—	—	0.031	6.43
Tenure^{2}2	—	—	—	—	−0.059	−4.24
Tenure3	—	—	—	—	0.028	5.51
Tenure^{2}3	—	—	—	—	−0.065	−3.66
Tenure4	—	—	—	—	0.030	9.38
Tenure^{2}4	—	—	—	—	−0.061	−6.43
S_v	0.347	22.96	0.336	24.02	0.335	23.67
S_u	0.313	43.51	0.299	45.29	0.299	45.10
Expected Value of e_v	−0.277		−0.269		−0.267	
Log-Likelihood	−2120		−1917		−1913	
OLS Log-Likelihood	−2146		−1947		−1943	
N	4825		4825		4825	

T-Values computed from standard errors obtained from the covariance of analytic first derivatives.

OLS Log-Likelihood is the Log-Likelihood value of the OLS estimates of the equation ($S_v = 0$) that were used to obtain starting values.

The expected value of $e_v = -(2^{1/2}/\pi^{1/2})S_v$ from [2].

The omitted occupational class dummy is for technical workers.

(1) a hedonic form with only the human capital variables: age (in years), age squared over 100, education (in years), tenure (years at current job), tenure squared over 100, and race (= 1 for whites, = 0 otherwise);
(2) a reduced form version including the human capital variables and controls for plant size (plant size = 1, if more than 100 employees, = 0 otherwise), and occupation. Four occupational classes are included: Executive-Professional-Managerial, Technical-Administrative, Service, and Operatives-Laborers-Production;
(3) a reduced form version including the human capital variables, the previous controls, and interaction terms between occupation and the tenure variables. (Each occupation has an associated tenured and tenured squared term.)

Table II. Returns to Tenure by Occupation

Years	Services	Executive	Operatives	Technical
A. Percent Difference from the No-Tenure Wage for Technicians				
0	−0.126	0.128	−0.089	0.000
1	0.036	0.030	0.027	0.030
5	0.164	0.137	0.124	0.137
10	0.292	0.245	0.216	0.244
15	0.378	0.323	0.275	0.320
20	0.424	0.372	0.302	0.366
B. Tenure-Earnings Profile in Dollars per Hour				
0	$4.37	$5.64	$4.55	$5.00
1	4.53	5.81	4.68	5.15
5	5.09	6.41	5.12	5.69
10	5.65	7.02	5.54	6.22
15	6.02	7.46	5.81	6.60
20	6.22	7.74	5.93	6.83

Computed by the authors from the estimates in Table I.

IV. Results

Table I presents the estimates from the three models. In each case the signs on the human capital variables have the correct sign and are significant. Hourly earnings increase about 7 percent for each year of education obtained and about three percent for each year of tenure at the current job. Each year of age increases hourly earnings about three percent. In the models including the plant size dummy variables the larger plant size increases hourly earnings by 20 percent. In each case the estimated variance of the half-normal term is similar in magnitude to the normal error term. The expected value of the half-normal term is close to negative 26 percent. This means that on average female hourly earnings are reduced by a maximum of 26 percent from the frontier. This estimate of direct discrimination against females is lower than those reported by Low and Villegas [7] who report estimates of unexplained male/female differentials of 55 percent in their hedonic model and 38 and 35 percent in their reduced form model. Other estimates of the differential they report range from 43 to 18 percent. Our results are quite consistent with those of Goldin and Polachek [5]. They show that accounting for expected human capital reduces the male-female earnings gap by close to 80 percent. If the original unexplained differential were 55 percent, adjusting for expected human capital would leave an 11 percent gap. This is closer to our estimated level of direct discrimination than to the original estimate.

Model 3 introduces occupation and tenure interaction terms to control in part for heterogeneity of the female workers[8]. These results clearly indicate that tenure-earnings profiles differ greatly across occupational groups. Table II demonstrates these differences. In spite of the model changes the estimate for s_v is reduced by less than 0.02. This would seem to indicate that additional controls do not change the basic error structure and that the model is robust.

These estimates of discrimination do not account for discrimination that may shape the occu-

8. Clearly to control entirely for female heterogeneity would be quite difficult and would require interaction terms for education and field of study and controls for industry.

pational, industrial, or educational choices of females, or the part-time versus full-time decision, but rather is an estimate of the difference between female earnings and female productivity.

V. Conclusion

This paper has presented a methodology for estimating discrimination against females that does not require assuming that all unmeasured male and female characteristics are identical. Direct discrimination is defined to be the difference between earnings and a stochastic earnings frontier. Estimates made using 1983 CPS data indicate that full-time, non-union female hourly earnings would be up to 25 percent higher in the absence of direct discrimination, assuming no labor market inefficiency. This estimate is lower than those made with traditional residual analysis, but cannot be subjected to similar criticism. This leads to two important conclusions that have a bearing on policy and further research. Even when eliminating all assumptions that males and females are identical it is not possible to eliminate the possibility of direct wage discrimination against females. Our results also imply that 25 to 50 percent of unexplained male-female differential estimated in traditional studies is due to differences between males and females. Our understanding of discrimination might be increased most appropriately by attempts to measure the extent of indirect discrimination, rather than in more refinements to traditional unexplained differential models which capture an undetermined mix of direct and indirect discrimination.

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