Energy Storage and Current Sheets in a Quadrupolar Corona

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Abstract

The energy that powers eruptive solar events such as coronal mass ejections is likely stored in non-potential coronal magnetic fields, where it builds up gradually but is then released quickly in the eruptive event itself. Here we explore the buildup of magnetic energy in a quadrupolar corona, a configuration that may characterize the corona at certain times between solar minimum and maximum, and more generally that permits theoretical exploration of magnetic-field evolution on length scales smaller than those of the dipole and related bipolar fields. We consider shearing the magnetic footpoints of one lobe or both lobes of the quadrupolar field, introducing force-free currents and additional magnetic energy associated with the changing magnetic-field configuration and especially with the presence of an azimuthal field component. Magnetic energy builds readily to levels that modestly exceed those possible with a pure dipole field. We compare the maximum possible stored energy with that of a quadrupolar field with open field lines (a non-trivial calculation, in contrast to the dipole case). For the pure dipole we earlier found energies some 10% above that of the open field; with the quadrupole we achieve nearly twice that.

Introduction

This work expands our earlier exploration of the buildup of magnetic energy in dipolar and related bipolar coronal magnetic field configurations (Wolfson et al. 2007; Wolfson & Pathak 2007). There we found force-free fields containing magnetic flux ropes, capable of storing sufficient energy to propel CMEs even after accounting for the energy needed to open the field.

Here we turn to quadrupolar fields, which provide a higher level of magnetic complexity and smaller scale lengths, and which approximate the coronal field during certain times in the solar cycle (McIntosh 1993). Again we use the force-free approximation, appropriate in most of the lower corona. We build energy in our model corona by adding an azimuthal field component—equivalent physically to shearing the magnetic footpoints at the coronal base. The quadrupolar field has two distinct magnetic lobes, one in each hemisphere, and we can shear the field in one hemisphere or both. The former case permits exploration of the interaction between a sheared field and an adjacent unsheared flux system—something that commonly happens in the evolution of the coronal field toward and through the CME process.

Normally, the energy in a sheared force-free magnetic field cannot exceed that of a fully open field with the same boundary conditions (Aly 1984, 1991; Sturrock 1991). This is problematic for CME energy storage because the CME requires not only that the field open but also that there be sufficient excess energy to lift and accelerate the ejected material. Our previous work overcomes this issue through the formation of detached magnetic flux ropes, to which the Aly-Sturrock limit does not apply. In the present work the limit also does not apply to quadrupolar fields sheared asymmetrically in a single hemisphere, where only a partial opening of the field is necessary.

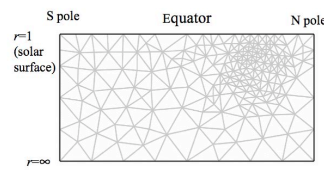

Fig. 1 Computational domain, showing a typical finite-element mesh after the first pass of adaptive refinement, for the asymmetrically sheared quadrupole.

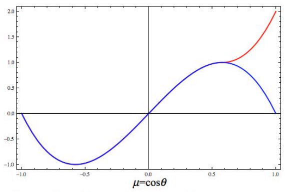

Fig. 2 Boundary condition on ψ for a pure quadrupole (blue) and as modified for a potential field open in the northern hemisphere (red).

Computational Technique

We work in axisymmetric spherical geometry. With one ignorable coordinate, Ampère's law $(\mathbf{J}\times\mathbf{B}=\mathbf{0})$ and the force-free condition $(\mathbf{J}\propto\nabla\times\mathbf{B})$ combine to give a Grad-Shafranov equation, which we write symbolically in the form

$$L(\psi)=\gamma^2 f(\psi)\frac{df}{d\psi} \qquad (1)$$

Here L is a linear partial differential operator and ψ is the dependent variable, which in axisymmetric spherical geometry is given by $\psi=A_\phi r\sin\theta$ with A_ϕ the azimuthal component of the magnetic vector potential and r, θ the spherical polar coordinates. The function f is an arbitrary function of ψ, related directly to the azimuthal magnetic field component and indirectly to the magnetic footpoint shear. We fix the form of f and vary the parameter γ to change the function's overall amplitude.

Our goal is to find solutions with large magnetic energy—in particular, energy in excess of the Aly-Sturrock open-field limit. We specify a sequence of solutions in terms of the azimuthal magnetic flux as described in Wolfson et al. (2007). This approach helps us follow the sequence through critical points at which γ switches from increasing to decreasing or vice versa.

The numerical solution is done on a rectangular domain with coordinates $\mu=\cos\theta$ and $w=1/r$, using the finite-element package Comsol Multiphysics with adaptive meshing (Fig. 1). We then define extrusion variables that allow us to plot the results in spherical geometry.

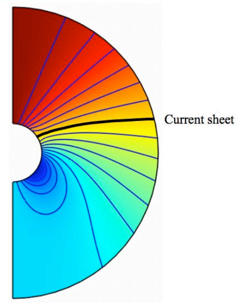

Fig. 3 Solution for a quadrupole with one lobe open, shown to 5 solar radii. Color codes the value of the flux function ψ, from –1 (blue) to 2 (red); contours of ψ trace the field lines. ψ=1 contour (black) marks where a current sheet would form in the corresponding non-potential field.

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Open-field Energy

Whether a model corona contains enough energy to power a CME depends on the total magnetic energy compared with that needed to open the field. Energy in excess of the field-opening energy is necessary to lift the ejecta against solar gravity and to propel it to CME speeds of typically 400 km·s^{-1}. So for any coronal model, we need to know the energy of the open magnetic field with the same boundary conditions at the coronal base.

Opening the coronal field entails the formation of one or more current sheets. For bipolar fields symmetric about the equator, the result is a single current sheet in the equatorial plane. With the current-sheet configuration thus known, it is straightforward to calculate the open-field energy (see Wolfson 2003). But with quadrupolar fields the current-sheet configuration is not obvious *a priori*, and must be calculated.

Here we follow a suggestion of Schatten (2008), namely to solve a related problem in which the radial field at the coronal base is reversed on one side of a bipolar region that would normally open to include a current sheet. The resulting field is strictly potential everywhere, with no current sheet, but its field magnitude is everywhere the same as the corresponding open field containing a current sheet. Therefore its magnetic energy is also the same. This new field is unphysical because a nonzero magnetic flux emerges from the model Sun, but it remains a useful computational device. Figure 2 shows a modified quadrupolar boundary condition that generates this field.

We solve for the flux function ψ of Equation (1) with γ=0, here using the Legendre solution described in Wolfson & Gould (1985):

$$\psi=\sum_{\ell=0}^{\infty}c_\ell r^{-\ell}\frac{P_{\ell-1}(\mu)-P_{\ell+1}(\mu)}{2\ell+1} \qquad (2)$$

Requiring that the field line from one pole extend to infinity sets the constant c_0. An orthogonal function analysis then provides a recursion relation for c_ℓ in terms of $c_{\ell-2}$. Then c_1 is set by requiring that the boundary condition be satisfied at the point μ=0. That procedure requires iteration as the whole set of c_ℓs for odd ℓ is involved. With the coefficients determined, we differentiate the series solution (2) to get the magnetic field, and then integrate B^2 to find the magnetic energy. The result, for a series to ℓ=12, is an open-field energy 1.5 times that of the closed potential field for the case of a quadrupole with one lobe open. Fig. 3 shows the resulting open field, including the configuration of the current sheet in the corresponding non-potential field. For a quadrupole with both lobes opened, the open-field energy is 2.39 times the potential-field energy.

Results

We explore two distinct situations: magnetic shear applied symmetrically in both hemispheres and shear applied asymmetrically, in only the northern hemisphere. Fig. 4 shows an asymmetric solution in the computational domain, while Fig. 5 shows the maximum-energy solutions for both cases plotted in spherical geometry. Note that flux ropes form in each case. Fig. 6 shows the solution sequences generated as the parameter γ in Equation (1) varies; we are able to converge solutions until just past the maximum energy. For the asymmetric case that maximum is 1.69 times the potential-field energy; for the symmetric case the corresponding ratio is 2.71. Although two-hemisphere shearing yields more energy, it suggests the unrealistic situation of two identical CME precursors developing simultaneously. In any event, it is the excess over the open-field energy that is available to drive a CME. Figure 7 compares these energies for the two cases, as well as for a the dipole case treated in our earlier work; the excess energies plotted are 19%, 32%, and 10% of the potential field-energy for, respectively, the asymmetrically sheared quadrupole, the symmetrically sheared quadrupole, and the dipole. Since the unrealistic case of the symmetrically sheared quadrupole should yield two distinct CMEs, the asymmetrically sheared quadrupole actually has the most excess energy on a per-CME basis—and nearly twice that of the dipole. Another significant comparison is with the recent work of Zhang & Flyer (2008), who model multipolar force-free fields but without the overlying potential field that characterizes our work and that we believe is responsible for "holding down" the sheared field and thus allowing greater energy buildup. Indeed, our model achieves twice the free energy of Zhang & Flyer's model.

We conclude that the quadrupole, with its smaller spatial scale, yields considerably more CME-powering excess energy than the larger-scale dipole, and that the overlying potential field continues to playsa role in this energy buildup. This result is consistent with our earlier findings about the role of flux distribution in the energy buildup in purely bipolar fields (Wolfson 2003). At this point we have not applied the energy-maximizing procedure described in Wolfson et al. (2007), so we anticipate that optimized quadrupolar fields might store even greater energies than those found here.

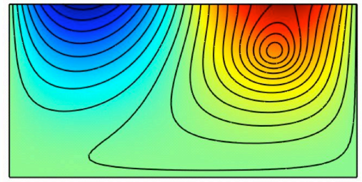

Fig. 4 Maximum-energy solution for one sheared lobe, in the computational domain that extends from the solar surface (top) to infinity (bottom); see Fig. 1. Contours are projections of the magnetic field lines in the r-θ plane.

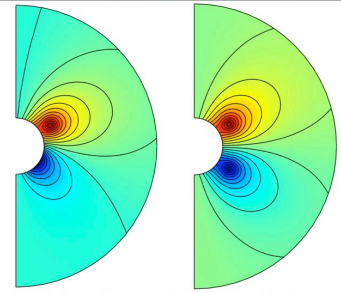

Fig. 5 Projections of magnetic field lines in the r-θ plane for asymmetric (left; one lobe sheared) and symmetric (right) shear, for the maximum-energy solutions. Colors code values of the flux function ψ. Note the two flux ropes at right and one at left.

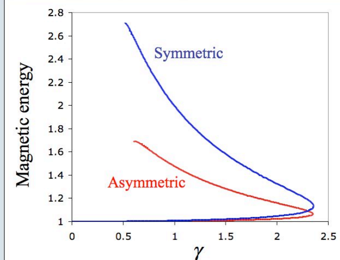

Fig. 6 Energy buildup along solution sequences with symmetric and asymmetric shear. Energy is in units of the potential-field energy.

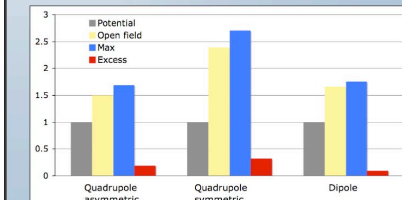

Fig. 7 Characteristic energies for three field configurations. Excess energy is available to lift and accelerate CMEs. Potential-field energies are normalized to 1.