Algebra: Laws of Operation

Arithmetic Operations

Let a, b, c, and d be real numbers. Then

$a(b+c) = ab + ac$

$(b+c)a = ba + ca$

$\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b}$, $(b \neq 0)$.

$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$, $(b, d \neq 0)$.

$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$, $(b, c, d \neq 0)$.

Caution! Common Mistakes to Avoid

$5(a+3) = 5a + 3 \ldots$ Oops! Should be $5a + 15$.

$-4(x-2) = -4x - 8 \ldots$ Wait! Should be $-4x + 8$.

$\frac{6x+1}{x} = 6 + 1 = 7 \ldots$ Oops! Should be $6 + \frac{1}{x}$.

$\frac{3}{3x+1} = \frac{1}{x} + 3 \ldots$ Wait! Invalid operation!

Exponents and Radicals

Let a and b be positive real numbers, and let r and s be rational numbers (that is, quotients of integers.) Then

$a^r \times a^s = a^{r+s}$

$\frac{a^r}{a^s} = a^{r-s}$

$(a^r)^s = a^{rs}$

$(ab)^r = a^r b^r$

$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$

$a^{-r} = \frac{1}{a^r}$

$\left(\frac{a}{b}\right)^{-r} = \frac{b^r}{a^r}$

Now let n and m be positive integers. Then

$a^{1/n} = \sqrt[n]{a}$

$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$