We can also estimate the result of a calculation. Glaciologists have determined the average width of the Little Knifepoint Glacier as 382 meters. Assuming a rectangular shape, the glaciologists then calculated the area of the glacial ice:

$$\text{area} = \text{length} \times \text{average width} = 561\text{ m} \times 382\text{ m} = 214{,}302\text{ m}^2$$

The area of the Little Knifepoint Glacier is 214,302 square meters. Is this number reasonable? To make an estimate of the calculation, first round off the two distances:

$$\text{area} = 561\text{ m} \times 382\text{ m} \approx 500\text{ m} \times 400\text{ m}$$

We rounded the length down and rounded the width up. Rounding some numbers up and some numbers down should give us a better estimate.

$$\text{area} \approx 500\text{ m} \times 400\text{ m} = 200{,}000\text{ m}^2$$

The Little Knifepoint Glacier covers about 200 thousand square meters of area. The estimate is very close to the actual value of 214,302 square meters and provides a quick check on the detailed calculation.

It's very common in environmental science to make "back of the envelope" estimates for many types of questions. For example, how do we estimate the volume of water an average household uses each year to wash dishes?

Let's assume an average household washes dishes once a day and that it uses one sinkfull of water for washing and rinsing. We can mentally compare the volume of the sink with a gallon of milk (for example) to estimate that a sink holds about 6 to 10 gallons of water:

$$\text{gallons per year} = 6\text{ to }10\text{ gallons per day} \times 365\text{ days per year}$$

The calculation can be rounded off for this estimate:

$$\begin{aligned}\text{gallons per year} &\approx 8\text{ gallons per day} \times 400\text{ days per year}\\ &= 3{,}200\text{ gallons per year}\end{aligned}$$

The "back of the envelope" estimate of water use for dishwashing is about 3,000 gallons of water per year per household.

UNITS OF MEASUREMENT

Units are the conventions or "yardsticks" of measurement for various quantities such as dimension and size. For example, distance can be measured in familiar units such as millimeters, inches, feet, meters, and miles. Tables 1-2 through 1-4 give a few common units of measurement. There is a larger and more explanatory table of units in the Appendix at the back of this book; take a quick glance at it now. Tables 1-2 through 1-4 also include abbreviations for the different units listed. It is useful to memorize a few key abbreviations and to watch out for similar-appearing units; for example, m is meter, but mi is mile.

TABLE 1-2 Metric Units

Distance	millimeter (mm)	meter (m)
Area	square meter (m^2)	hectare (ha)
Volume	cubic cm (cc or cm^3)	liter (l or L)
Mass	gram (g or gm)	tonne
Energy	joule (J)	kilojoule (kJ)
Power	watt (W)	gigawatt (GW)
Heat	calorie (cal)	kilocalorie (Cal)

TABLE 1-3 English Units

Distance	inch (in)	mile (mi)
Area	acre	square mile (mi^2)
Volume	quart (qt)	gallon (gal)
Mass	slug (no kidding)	
Energy	foot-pound (ft-lb)	erg
Power	ft-lb/sec	horsepower (hp)
Heat	British thermal unit (BTU)	

TABLE 1-4

Time	second (sec or s)	minute (min)	year (yr or a)
Temperature	Celsius (° C)	Fahrenheit (° F)	Kelvin (K)

Most scientists use the metric system. In this text, you will have plenty of opportunity to use metric units; however, we will provide both metric and English units as often as is reasonable.

There are a few simple rules for the arithmetic of units. Units that are the same can be added or subtracted without changing the unit. For example,

$$12\text{ centimeters} + 9\text{ centimeters} = 21\text{ centimeters}$$

$$4\text{ quarts} - 3\text{ quarts} = 1\text{ quart}$$

When multiplying and dividing the same units, treat the units like numbers. For example,

$$3\text{ feet} \times 3\text{ feet} = \underbrace{3 \times 3}_{\text{numbers}}\ \underbrace{\text{feet} \times \text{feet}}_{\text{units}} = 3^2\text{ feet}^2 = 9\text{ feet}^2$$

$$\frac{10\text{ cm}^3}{2\text{ cm}^2} = \frac{10}{2}\frac{\text{cm}^3}{\text{cm}^2} = 5\frac{\text{cm}\times\cancel{\text{cm}^2}}{\cancel{\text{cm}^2}} = 5\text{ cm}$$

Units can cancel completely during division. For example,

$$\frac{6\text{ miles}^2}{12\text{ miles}^2} = \frac{6}{12}\frac{\text{miles}^2}{\text{miles}^2} = 0.5\frac{\cancel{\text{miles}^2}}{\cancel{\text{miles}^2}} = 0.5$$

This last result has no units.

UNIT CONVERSION

The Amazon River in South America is approximately 3,900 miles long. How many kilometers is that? In order to solve this problem, we need the **conversion identity** that relates miles to kilometers. A few common identities are given in Tables 1-5 through 1-7. More extensive tables are found in the Appendix.

TABLE 1-5 Metric to Metric

1 cm	=	10 mm	1 cm^3	=	1,000 mm^3
1 meter	=	100 cm	1 liter	=	1,000 cm^3
1 kilometer	=	1,000 m	1 m^3	=	1,000 liters
1 m^2	=	10,000 cm^2	1 kilogram	=	1,000 grams
1 hectare	=	10,000 m^2	1 tonne	=	1,000 kg
1 km^2	=	1,000,000 m^2	1 km^2	=	100 hectare

TABLE 1-6 English to English

1 foot	= 12 inches	1 quart	= 2 pints
1 yard	= 3 feet	1 quart	= 57.75 in^3
1 mile	= 5,280 feet	1 gallon	= 4 quarts
1 acre	= 43,560 ft^2	1 ft^3	= 7.4805 gallons
1 $mile^2$	= 640 acres	1 pound	= 16 ounces
1 quart	= 4 cups	1 ton (short)	= 2,000 pounds

TABLE 1-7 English to Metric

1 inch	= 2.54 cm	1 $mile^2$	= 2.59 km^2
1 foot	= 0.3048 m	1 quart	= 0.9464 liters
1 yard	= 0.9144 m	1 gallon	= 3.7854 liters
1 mile	= 1.609 km	1 pound	= 0.4536 kg
1 acre	= 4,046.9 m^2	1 ton (short)	= 0.9072 tonnes
1 acre	= 0.4047 ha	1 BTU	= 251.996 calories

From Table 1-7, we can see that 1 mile equals 1.609 kilometers (approximately). Because "1 mile" and "1.609 km" are equivalent, we can simply substitute for the length of the Amazon River in miles and evaluate:

$$3{,}900 \text{ mi} = 3{,}900\,(1 \text{ mi}) \approx 3{,}900\,(1.609 \text{ km}) \approx 6{,}300 \text{ km}$$

The previous unit change is the easiest kind of unit conversion problem; just substitute the equivalent unit and simplify to get the desired result. We refer to this approach as the **substitution method** of unit conversion.

Figure 1-5 Oil from the 2002 *Prestige* spill. This heavy oil did not spread out as a thin film. Liencres, Cantabria, Spain. *Photo by Eduardo Cando (www.costaquebrada.com).*

EXAMPLE 1-4 *Prestige* **Oil Spill Volume**

In November 2002, the 25-year-old, single-hulled tanker *Prestige* broke in half off the Spanish coast (Figure 1-5).[3] This tanker was carrying about 19.6 million gallons of heavy fuel oil. How many liters of oil is this?

Solution From the conversion tables, we see that 1 gallon is 4 quarts, and 1 quart is approximately 0.9464 liters. We first convert gallons to quarts:

$$19.6 \text{ million gallons} = 19.6 \text{ million}\,(4 \text{ quarts}) = 78.4 \text{ million quarts}$$

We then convert quarts to liters:

$$78.4 \text{ million quarts} = 78.4 \text{ million}\,(0.9464 \text{ liters}) \approx 74.198 \text{ million liters}$$

Therefore, 19.6 million gallons of oil is about 74.2 million liters.

Many conversion problems are *slightly* more complicated because there is no ready equivalency in a table. For example, suppose the average spacing between old-growth mahogany trees in a tropical rainforest is 36 meters. How many feet is this? From the conversion tables, we find that 1 foot = 0.3048 meters. This equivalency is useful but it is not in the right form for this problem. Therefore, we divide both sides by 0.3048, and simplify:

$$1 \text{ foot} = 0.3048 \text{ meters}$$

$$\frac{1 \text{ foot}}{0.3048} = \frac{0.3048 \text{ meter}}{0.3048}$$

$$3.281 \text{ feet} = 1 \text{ meter}$$

This new equivalency looks correct; 1 meter is a bit longer than 1 yard, which is 3 feet. We can now substitute this equivalency to get the spacing between old mahogany trees in feet:

$$36 \text{ meters} = 36\,(3.281 \text{ feet}) \approx 118 \text{ feet}$$

EXAMPLE 1-5 *Prestige* **Oil Spill Thickness**

If all the fuel oil from the *Prestige* spilled out onto the surface in a layer 4 inches thick, the resulting oil slick would cover about 741,946 square meters in area. How many acres is this?

Solution From Table 1-7, 1 acre = 4,046.9 m^2. But we're converting a known number of square meters to an unknown number of acres. We need to rearrange the identity such that 1 m^2 = *xxx* acres. We divide both sides of the equality by 4,046.9, and simplify:

$$1 \text{ acre} = 4{,}046.9 \text{ m}^2$$

$$\frac{1 \text{ acre}}{4{,}046.9} = \frac{4{,}046.9 \text{ m}^2}{4{,}046.9}$$

$$0.0002471 \text{ acres} = 1 \text{ m}^2$$

This last result is a new equality in a form that is appropriate for the substitution method of unit conversion. Now substitute this identity to convert square meters to acres:

$$741{,}946\,(\text{m}^2) = 741{,}946\,(0.0002471 \text{ acres}) \approx 183.3 \text{ acres}$$

The fuel oil carried by the *Prestige* would cover 183 acres (about a third of a square mile) to a depth of 4 inches.

Another common approach to unit conversion uses **conversion ratios**. The equivalency is expressed as a ratio of the two related units. This ratio is equal to 1 and therefore can be substituted readily into equations. For example, there are two conversion ratios between feet and meters:

$$\frac{1 \text{ foot}}{0.3048 \text{ meters}} = 1 \quad \text{or} \quad \frac{0.3048 \text{ meters}}{1 \text{ foot}} = 1$$

Let us try the mahogany spacing problem again, using a conversion ratio. We start with a 36-meter spacing and use the first of the two conversion ratios:

$$36 \text{ meters} = 36 \text{ m} \times \frac{1 \text{ ft}}{0.3048 \text{ m}} = \frac{36 \text{ m} \times 1 \text{ ft}}{0.3048 \text{ m}} = \frac{36}{0.3048}\,\frac{\cancel{\text{m}} \times \text{ft}}{\cancel{\text{m}}} \approx 118 \text{ ft}$$

Once again we find that the mahogany trees are spaced about 118 feet apart.

The conversion ratio used in the mahogany problem can be expressed as feet per meter, or meters per foot. How do we know which ratio to use? There are two hints to assist you: (1) The units should cancel, and (2) The numbers should make sense.

1. *Watch the units.* We start with meters and want to convert to feet; the meters must cancel out. Therefore, multiply by the conversion factor that has meters in the denominator:

$$\text{meters} = \text{meters} \times \frac{\text{feet}}{\text{meters}} = \frac{\cancel{\text{meters}} \times \text{feet}}{\cancel{\text{meters}}} = \text{feet}$$

   Notice how the meters in the numerator and denominator cancel out.

2. *Should the number go up or down?* Meters are larger units of distance than feet. When converting from meters to feet, the number of feet should be greater than the number of meters. Therefore, multiply by the conversion ratio that makes the number increase:

$$36 \text{ meters} \times \frac{1 \text{ foot}}{0.3048 \text{ meters}} \approx 118 \text{ feet}$$

EXAMPLE 1-6 Hectares in a Section of Land

A **section** of land in the section-township-range system used in the United States is 1 square mile in area. How many metric hectares are there in a section?

Solution Using the tables, we see that there are 640 acres in one square mile and that there are approximately 2.47 acres in one hectare. Therefore,

$$1 \text{ mile}^2 = 640 \text{ acres} = 640 \text{ acres} \times \frac{1 \text{ hectare}}{2.47 \text{ acres}} \approx 259 \text{ hectares}$$

The first part of this solution is a simple identity ($1 \text{ mile}^2 = 640$ acres). The second part involves multiplying by the correct conversion ratio. We may not know, in advance, whether the number of hectares should be smaller or larger than the number of acres. But we do know that the acres must cancel, leaving hectares:

$$\text{acres} = \text{acres} \times \frac{\text{hectare}}{\text{acres}} = \cancel{\text{acres}} \times \frac{\text{hectare}}{\cancel{\text{acres}}} \approx \text{hectares}$$

COMPOUND UNITS

Basic units of time, distance, mass, and so on can be combined to produce **compound units** for quantities such as velocity, density, and energy. For example, typical units of velocity are miles per hour (mi/hr), a unit of distance divided by a unit of time. Another unit of velocity is kilometers per second (km/sec).

Converting one compound unit to another is straightforward but can involve a large number of steps. For example, to convert miles per hour to kilometers per second, we have to convert both the distance in the numerator (miles to kilometers) and the time in the denominator (hours to seconds):

$$70\,\frac{\text{mi}}{\text{hr}} = 70\,\frac{\text{mi}}{\text{hr}} \times \left(\frac{1.6 \text{ km}}{1 \text{ mi}}\right) \times \left(\frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}}\right) \approx 0.031\,\frac{\text{km}}{\text{sec}}$$

Study this example to confirm that the units cancel correctly to yield $\frac{\text{km}}{\text{sec}}$. Also check whether the number makes sense. Seventy miles in one hour should be a small distance in one second.

We also recommend writing these conversions with the units and values "stacked" on top of each other, as shown in the preceding examples. For example, we recommend $\frac{50 \text{ gallons}}{6 \text{ minutes}}$, not 50 gallons/6 minutes. When the units and values are written in a "stack," it is easy to see how the units cancel, and it is easy to see how to arrange the conversion ratios. Make sure the units cancel properly and check the result.

Another recommendation for avoiding mistakes in a complicated unit conversion is to arrange the numbers on the left and units on the right. This recommendation is illustrated in Example 1-7.

EXAMPLE 1-7 Acre-Feet of Water

A cotton farmer applied 2 acre-feet of irrigation water to his cotton crop. How many cubic feet is this volume?

Solution The first challenge is to recognize that the unit "acre-foot" means "acre times foot." The hyphen is often used instead of a multiplication symbol in this particular compound unit. A unit of area times a unit of length is a volume. To convert acre-feet to cubic feet, we must know the number of square feet in an acre. From Table 1-6 we see that

$$1 \text{ acre} = 43{,}560 \text{ ft}^2$$

This equivalency is shown in Figure 1-6.

We can substitute this equality to change acres to square feet:

$$2 \text{ acre-feet} = 2 \text{ acre} \times 1 \text{ foot} = 2(43{,}560 \text{ ft}^2) \times 1 \text{ foot}$$

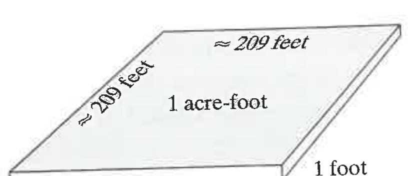

Figure 1-6 1 acre-foot of volume. The 1-foot depth has been exaggerated.

We can rewrite the rightmost portion of the previous calculation so that the numbers are on the left and the units are on the right:

$$2\,(43{,}560 \text{ ft}^2) \times 1 \text{ foot} = \underbrace{2 \times 43{,}560 \times 1}_{\text{numbers together}} \quad \underbrace{\text{ft}^2 \times \text{ft}}_{\text{units together}} = 87{,}120 \text{ ft}^3$$

There are 87,120 ft^3 of water in 2 acre-feet of water. Rearranging the computation with the numbers on the left and the units on the right can help with complicated conversions.

Converting compound units can be tedious, but these conversions arise all the time. In 2002, the U.S. Coast Guard seized a fishing vessel carrying 32 tons of shark fins, destined for the Asian soup market.[4] Sharks are a very important part of marine ecosystems, yet they are routinely killed just for the fins. How many sharks were killed to produce 32 tons of fins? An average shark might have 1.5 pounds of fin. Using this estimate, we can convert tons of shark fins to number of sharks in two steps:

$$32 \text{ tons of shark fins} \times \frac{2{,}000 \text{ pounds}}{1 \text{ ton}} = 64{,}000 \text{ pounds of shark fins}$$

$$64{,}000 \text{ pounds of shark fins} \times \frac{1 \text{ shark}}{1.5 \text{ pounds of shark fins}} \approx 43{,}000 \text{ sharks}$$

Over 40,000 sharks were killed just for their fins. The rest of the body parts were tossed overboard.

UNITS IN EQUATIONS AND FORMULAS

Units are sometimes more important than numbers in understanding reality-based mathematics problems and getting the computations right. Here is an example where units aid hydrologists studying stream flow.

EXAMPLE 1-8 Owen's River Discharge

The size of a river is often characterized by its **discharge**, the volume of water passing by a measuring point on the river in a certain amount of time. Units of discharge might be gallons per minute, cubic feet per second, or cubic meters per second. Imagine standing in a small stream, with a 5-gallon bucket in hand, trying to scoop out all of the water passing by every second. This cannot be done, even with a tiny stream.

The Owens River originates on the east side of the Sierra Nevada near the California-Nevada border. A 200-mile-long aqueduct brings the river water around the mountains to Los Angeles (Figure 1-7). In 1884, J. M. Keeler declared that

Figure 1-7 The Owens River is diverted through a large pipe to Los Angeles. *Courtesy of Eric and Avery Wilmanns.*

the "Owens River carries a volume of water, carefully measured, fifty feet wide, average depth six feet, flow [7 feet per second]; and . . . will by irrigation give to the county 50,000 acres of fine agricultural land."[5] How is the discharge of the Owens River determined using this information?

Solution Discharge cannot be measured directly but can be *calculated* from other measurements. The measurements of width, depth, and velocity can be combined to calculate discharge:

$$\text{width} \times \text{average depth} \times \text{average velocity}$$

We can check the mathematics with English units:

$$\text{feet} \times \text{feet} \times \frac{\text{feet}}{\text{sec}} = \frac{\text{ft}^3}{\text{sec}}$$

The result is volume per time. Unit analysis shows that discharge can be calculated by multiplying width, average depth, and average velocity. The discharge of the Owens River in 1884 was

$$50\text{ feet} \times 6\text{ feet} \times \frac{7\text{ feet}}{\text{sec}} = \frac{2{,}100\text{ ft}^3}{\text{sec}}$$

You will see more examples of units in equations in Chapter 4 on linear functions.

UNIT PREFIXES

The Darlington nuclear power facility in Ontario Province, Canada, can produce up to 3,524,000,000 watts of power when operating at peak capacity.[6] Cryptosporidium is a microbe that can contaminate municipal water supplies. Cryptosporidium is usually about $\frac{6}{1{,}000{,}000}$ of a meter in length; a very fine mesh must be used to filter out this microbe.

These very large and very small numbers can be difficult to convey in writing and speech and can easily lead to mathematical errors during computation. To simplify these numbers, scientists and others use **unit prefixes** (Table 1-8). A larger table of unit prefixes is given in the Appendix.

TABLE 1-8

Prefix	Symbol	Value	Exponential Equivalent	Example Using Prefix
giga	G	1 billion	10^9	gigawatt (GW)
mega	M	1 million	10^6	megayear (Ma)
kilo	k	1 thousand	10^3	kilopascal (kPa)
centi	c	1 hundredth	10^{-2}	centimeter (cm)
milli	m	1 thousandth	10^{-3}	milliliter (mL)
micro	μ	1 millionth	10^{-6}	micrometer (μm)

The Darlington nuclear facility generates 3.524 billion watts or 3.524 gigawatts of electricity, abbreviated 3.524 GW. Cryptosporidium is usually about $\frac{6}{1{,}000{,}000}$ of a meter in length, or 6 one-millionths of a meter, or 6 micrometers, abbreviated 6 μm.

The prefix **milli** is abbreviated "m," which is also the abbreviation for **meter.** "Milli" is a prefix and so will always precede another unit. A milliliter (ml or mL) is a thousandth of a liter, and a millimeter (mm) is a thousandth of a meter.

As can be seen in Table 1-8 and the Appendix, there's a prefix for some of the powers of 10 but certainly not all of them. With one exception (centi), the prefixes in Table 1-8 and the Appendix correspond to powers of 10 whose power or superscript is a multiple of 3.

EXAMPLE 1-9 Heat from Coal

Coal provides between 9,500 and 14,000 British thermal units (BTUs) of heat energy per pound. Express this range of values in calories. Then rewrite both the BTUs and calories using unit prefixes.

Solution There are 251.996 calories per BTU; therefore, the range of heat production for coal is 2,393,962 to 3,527,944 calories per pound. Coal generates 9.5 to 14 kiloBTUs or approximately 2.393 to 3.528 megacalories of energy per pound.

SCIENTIFIC NOTATION AND ORDER OF MAGNITUDE

The Darlington nuclear power facility produces about 3,524,000,000 watts of electricity at peak capacity, or 3.524 billion watts. The word *billion* represents the number 1,000,000,000, which can be written as 10^9. In other words, the Darlington nuclear power facility can produce up to 3.524×10^9 watts of electricity. Numbers expressed in **scientific notation** have a decimal value between 1 and 10 multiplied by 10 raised to some integer power. Some examples are 2×10^3, 5.7×10^{-3}, and 9.4×10^{11}.

EXAMPLE 1-10 Size of Cryptosporidium Cysts

Cryptosporidium cysts have a diameter of approximately 0.000006 meters or $\frac{6}{1{,}000{,}000}$ meters. What is that value in scientific notation?

Solution We know that 1 one-millionth is equal to 10^{-6}. Therefore, 6 one-millionths of a meter can be expressed as 6×10^{-6} meters.

USING TECHNOLOGY: SCIENTIFIC NOTATION

The size of a cryptosporidium cyst in scientific notation (6×10^{-6}) is typically entered into the calculator as $6 \times 10 \wedge -6$. Many calculators will have a shortcut key that can be used for scientific notation such as **E** or **EE** (e.g., 6 E −6), $\mathbf{10^x}$ (e.g., 6×10^{-6}), or $\boldsymbol{y^x}$, where 10 can be substituted for *y*. More details for the TI-83/84 graphing calculator are given next.

Number Display on the TI-83/84

Press **MODE** for three choices: **Normal** (typical display), **Sci** (scientific notation) and **Eng** (engineering notation). Using each choice, multiply 12,345 times 12,345. "E8" stands for "$\times 10^8$". The number of decimal places is controlled by the second line under **MODE**.

```
12345*12345
        152399025
12345*12345
     1.52399025E8
12345*12345
     152.399025E6
```

Using scientific notation, it is much easier to compare measurements or quantities when those quantities show a wide range of values. For example, imagine three different electricity generating plants of different sizes; 3,524,000,000 watts, 17,700,000 watts, and 5,500 watts (Table 1-9). Using scientific notation, we can express these three values as 3.524×10^9 watts, 1.77×10^7 watts and 5.5×10^3 watts. Scientific notation draws our attention to the power of 10 or **order of magnitude** of a number. The first electrical plant is approximately 2 orders of magnitude (10^2 times) greater in capacity than the second, and the second plant is a bit under 4 orders of magnitude (10^4 times) greater in capacity than the third.

TABLE 1-9

Plant Size	Scientific Notation
3,524,000,000 watts	3.524×10^9 watts
17,700,000 watts	1.77×10^7 watts
5,500 watts	5.5×10^3 watts

Scientific notation is also very useful in simplifying the arithmetic of very large or very small numbers.

EXAMPLE 1-11 U.S. Per Capita Energy Consumption, 2001

The United States consumed approximately 97,000,000,000,000,000 British thermal units of energy in 2001.[7] In the same year, the U.S. population was approximately 285,000,000 people.[8] What was the per capita energy consumption?

Solution Energy consumption and U.S. population in scientific notation are 9.7×10^{16} BTU and 2.85×10^8 people, respectively. Therefore, per capita energy use was

$$\frac{9.7 \times 10^{16}\,\text{BTU}}{2.85 \times 10^{8}\,\text{people}} = \frac{9.7}{2.85} \times 10^8 \frac{\text{BTU}}{\text{person}} \approx 3.40 \times 10^8 \frac{\text{BTU}}{\text{person}}$$

POWERS OF 10 AND LOGARITHMS

Not satisfied with scientific notation, mathematicians and scientists have developed an even more compact method of expressing large and small numbers. Let's return to the Darlington electrical facility, which has a 3,524,000,000-watt or 3.524×10^9–watt capacity. This generating capacity is somewhere between 10^9 and 10^{10} watts, some *noninteger power* of 10. What is that fractional power? In other words, what value of x makes $10^x = 3{,}524{,}000{,}000$?

To estimate the answer, we start by using the calculator to "guess and check," a process more formally known as **successive approximation**. Most scientific calculators will have a $\mathbf{10^x}$ key. We plug in some numbers between 9 and 10 and track the results (Table 1-10).

TABLE 1-10

1st guess	10^9	1,000,000,000
2nd guess	$10^{9.1}$	1,258,925,412
3rd guess	$10^{9.5}$	3,162,277,660
4th guess	$10^{9.55}$	3,548,133,892
5th guess	$10^{9.54}$	3,467,368,505
6th guess	$10^{9.545}$	3,507,518,740
7th guess	$10^{9.548}$	3,531,831,698
8th guess	$10^{9.547}$	3,523,708,710
9th guess	$10^{9.5471}$	3,524,520,168

With a few more guesses we find that

$$3{,}524{,}000{,}000 \text{ watts} \approx 10^{9.54704} \text{ watts}$$

The Darlington nuclear complex can produce about $10^{9.54704}$ watts of electricity.

There has to be a more direct way to get the power of 10, and there is: the **LOG** or $\mathbf{LOG_{10}}$ key on your calculator:

$$\log(3{,}524{,}000{,}000) \approx 9.54704$$

"Log" is an abbreviation for **logarithm**. In simple terms, the logarithm of a number gives the power of 10 that represents that number. In other words, the logarithm gives the precise order of magnitude based on 10. One hundred can be represented as 10^2, and so the logarithm of 100 is 2 (the power of 10 that makes 100).

$$\log(100) = \log(10^2) = 2$$

One one-hundredth $(\frac{1}{100})$ can be represented as 10^{-2}; therefore, the logarithm of 10^{-2} is -2:

$$\log\left(\frac{1}{100}\right) = \log(10^{-2}) = -2$$

Table 1-11 gives some numbers and their logarithms so you can see the pattern. Try computing these logarithms with your calculator.

TABLE 1-11

Number	Scientific Notation	Logarithm
1/1,000,000,000	10^{-9}	−9
1/1,000	10^{-3}	−3
1/10	10^{-1}	−1
1	10^0	0
10	10^1	1
10,000	10^4	4
1,000,000	10^6	6

It is assumed that the word *log* refers to logarithms based on 10. For this reason, log is often written as $\log_{10}$ and pronounced "log base 10." Anytime you see "log" it means $\log_{10}$, and vice versa.

Any positive real number can be a base of a log. Other logarithms include $\log_2$, $\log_7$, and $\log_e$, where e is Euler's constant (approximately 2.718281828). When working with logs that have a base not equal to 10, it is necessary to indicate the base using a subscript. These logs usually do not have their own keys on the calculator, with the exception of $\log_e$, which is typically the **LN, Ln,** or **ln** key on the calculator. The logarithms $\log_{10}$ and $\log_e$ are used so often that they have been given special names. Log_{10} is called the **common logarithm** and $\log_e$ is called the **natural logarithm**. As mentioned above, "log" means $\log_{10}$ and LN means $\log_e$. In this book we will be using $\log_{10}$ exclusively.

LOGARITHMIC SCALES

In the previous section, we learned how to represent a single value (3,524,000,000) by its logarithm (≈9.54704), the power of 10 that represents that number. When values of size range over many orders of magnitude, scientists may substitute the logarithms for the original numbers and focus on the differences in orders of *magnitude* rather than the differences among the numbers themselves. This approach creates a **logarithmic scale** of the values.

EXAMPLE 1-12 pH

One measure of the chemical composition of liquids is **pH**, which stands for the "power of hydrogen." pH is a logarithmic scale based on powers of 10 and is a measure of the concentration of hydrogen ions (H^+) in a liquid. In distilled water at a temperature of 25°C, there are 1.0×10^{-7} **moles** of H^+ in a liter. A mole is a very large number of ions or molecules (see the Appendix). By definition, a liquid with this concentration of hydrogen ions, 1.0×10^{-7} moles/liter, is said to be **neutral**.

To calculate the pH of a liquid, we take the negative of the logarithm of the H^+ concentration in moles/liter:

$$\text{pH} = -\log(\text{concentration of } H^+)$$

For example, the pH of a neutral liquid is 7 because

$$-\log(1.0 \times 10^{-7}\ \text{moles/liter}) = 7$$

Lemon juice has a much higher concentration of hydrogen ions than distilled water; 1.0×10^{-2} moles of H^+ per liter of liquid is a typical concentration in lemon juice. What is the pH of lemon juice?

$$\text{lemon juice pH} = -\log(1.0 \times 10^{-2}) = 2$$

Lemon juice has a pH of 2; liquids with low pH (that is, high concentrations of H^+) are **acidic** (Table 1-12).

TABLE 1-12

Liquid	Hydrogen Ions	pH	
Hydrochloric acid	1×10^{0} moles/liter	0.0	acidic
Lemon juice	1×10^{-2} moles/liter	2.0	acidic
Distilled water	1×10^{-7} moles/liter	7.0	neutral
Blood	4×10^{-8} moles/liter	7.4	basic
Ammonia	1×10^{-11} moles/liter	11.0	basic
Sodium hydroxide	1×10^{-14} moles/liter	14.0	basic

Normal human blood has a hydrogen concentration of about 4×10^{-8} moles/liter. Thus the pH of blood is

$$\text{blood pH} = -\log(4 \times 10^{-8}) = 7.4$$

Liquids with pH greater than 7 are **basic** in composition; ammonia has a pH of 11 (Table 1-12). The pH for common liquids ranges from a low of zero (1 mole/liter of hydrogen atoms, as found in pure hydrochloric acid) to a high of 14 (10^{-14} moles/liter of hydrogen atoms, as found in pure sodium hydroxide).

Each integer *increase* in pH corresponds to a 10-fold *decrease* in hydrogen concentration; a liquid with a pH of 8 is ten times more acidic than a liquid with a pH of 9. You can understand why scientists sometimes use a logarithmic scale as opposed to the plain old numbers. Common liquids from apple juice to household cleaners show an enormous range in hydrogen concentration; therefore, a logarithmic scale based on order of magnitude is much more convenient.

EXAMPLE 1-13 Sizes of Earthquakes

Earthquakes are the shaking of the ground caused by two blocks of rock moving past each other, relatively quickly, along a break in the rocks, called a **fault**. When blocks of rock move past each other, the irregular movement of the two blocks causes waves to radiate out in all directions. The more movement along the fault during an earthquake event, the bigger the waves produced and the more likely the ground shaking will cause significant damage. Earthquakes come in a wide variety of sizes, from the gigantic earthquake in Alaska in November 2002, which rattled the Trans-Alaska Pipeline, to tiny vibrations that can only be measured by ultrasensitive machines. Earthquakes measured by a typical set of earthquake recorders range over 9 orders of magnitude in size.

Scientists measure the *size* of an earthquake event today with something called the **moment**, which is the product of the rock strength, the area of the fault between the two blocks, and the average amount of movement or slip of one block past the other. But rather than reporting the moment or actual size of the earthquake, scientists report the **moment magnitude**, which uses the logarithm of the moment:

$$\text{moment magnitude} = \log\left(\frac{\text{moment}^{2/3}}{10^{10.7}}\right)$$

In the formula above, the moment has units of dyne-centimeters, which is a very small unit of force multiplied by a small unit of distance.

The January 17, 1995, Kobe earthquake in south-central Japan that killed over 5,000 people and caused \$200 billion of damage had a moment of 2.5×10^{26} dyne-centimeters.[9] The moment magnitude is therefore

$$\text{Kobe magnitude} = \log\left(\frac{(2.5 \times 10^{26})^{2/3}}{10^{10.7}}\right)$$

In a complicated expression like the one above, it's a good strategy to break the computation into steps. Start by entering and evaluating the numerator inside the large parentheses on the calculator. Remember to enclose the 2/3 power in parentheses. Then divide the result by $10^{10.7}$ and take the logarithm. The moment magnitude of the 1995 Kobe earthquake is 6.9. See the following calculator screen.

```
2.5E26^(2/3)
           3.96850263E17
Ans/10^(10.7)
             7918203.744
log(7918203.744)
             6.898626672
```

The November 3, 2002, Denali earthquake in central Alaska that rattled the oil pipeline had a moment of 8.0×10^{27} dyne-centimeters and a moment magnitude of 7.9.[10] The Denali earthquake was 32 times larger than the Kobe earthquake, but the magnitude between these two earthquakes differs by a value of only 1. Each unit of magnitude on the moment scale for earthquakes represents a 32-fold difference in size. A moment magnitude 6 earthquake is *not* twice as big as a moment magnitude 3 earthquake; it is 32^3 or 33,000 times larger.

CHAPTER SUMMARY

Measurements such as the weight of radioactive waste and the rate of immigration involve quantifying physical, chemical, and biological variables. Environmental scientists are always concerned about the accuracy and precision of their measurements. **Accuracy** refers to how close the measurement is to the actual value, whereas **precision** refers to the degree of detail or refinement of the measurement.

Units are the conventions of measurement; for example, distance can be measured in meters or inches. Units can be converted from metric to English or vice versa using **conversion identities** that relate one system to another. A mile of stream-side habitat is approximately 1.6 kilometers, for example. **Compound units** are created by multiplying or dividing one unit by another; for example, a whooping crane flying 30 miles per hour or a farmer applying an acre-foot of irrigation water.

Unit prefixes are employed in order to express large or small quantities in a more succinct form. A billion watts of electric power is a gigawatt of power, whereas a water filter with holes a millionth of a meter in diameter has micrometer-sized holes.

Another shorthand system uses **scientific notation** or **powers of 10**; in 2005, there were approximately 6.45×10^9 people on Earth. The size difference between two objects, based on powers of 10, is often called the **order of magnitude**. The rhinovirus (2×10^{-8} meters long) is two orders of magnitude smaller than the *E. coli* bacterium (2×10^{-6} meters). **Logarithms** can be used for precise calculation of powers of 10. The logarithm of 6.5×10^3 is 3.813; therefore, 6.5×10^3 hectares of nature preserve is equivalent to $10^{3.813}$ hectares of preserve.

When objects such as earthquakes or wildfires vary in size over many orders of magnitude, scientists may use a **logarithmic scale**, often based on powers of 10. For example, the pH scale is based on the logarithm of the hydrogen concentration; a liquid with a pH of 3 has 10 times the hydrogen concentration of a liquid with a pH of 4.

END of CHAPTER EXERCISES

Measuring

1. Measure the length (long axis) and width (perpendicular to length) of the butter clam shell shown in Figure 1-8. Alternatively, collect your own bivalve shell (clam, mussel, oyster, etc.) and measure its length and width. What problems did you encounter making these measurements?

Figure 1-8 Butter clam shell. *Photo: Langkamp/Hull.*

2. Measure the length (long axis) and width (perpendicular to the length) of the oak leaf shown in Figure 1-9. Alternatively, collect your own leaf and measure its length and width. What problems did you encounter making these measurements?

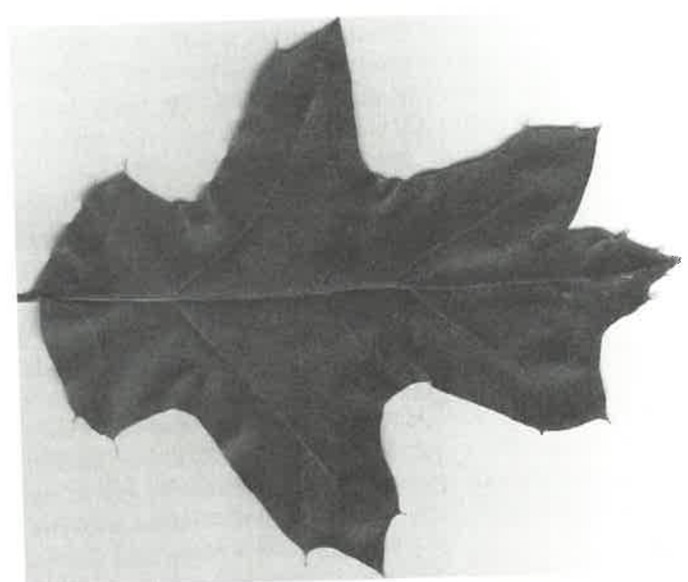

Figure 1-9 Red oak leaf. *Photo: Langkamp/Hull.*

Accuracy and Precision

3. The rate of evaporation of water from a reservoir was measured using microwave technology at 0.56 inches per day. The actual rate of evaporation was 0.49 inches per day. Discuss the accuracy of the microwave technology.
4. The thickness of soil in an agricultural plot was measured with ground-penetrating radar. The radar gave a thickness of 67 centimeters; the actual value, measured by

coring, was 126 centimeters. Discuss the accuracy of the ground-penetrating radar method.

5. The concentration of the gasoline additive methyl tert-butyl ether (MTBE) in groundwater beneath a leaking storage tank was measured at 3,144 parts of MTBE in a billion parts of groundwater (3,144 parts per billion parts). Discuss the precision of this measurement.
6. Researchers measured the concentration of water vapor in warm air using a laser system. The concentration of water vapor was found to be 28.3 grams of water vapor in 1,000 grams of air. Discuss the precision of this measurement.

Estimation

7. Estimate the gallons of gasoline consumed per year by a typical U.S. driver. How many miles are driven per year? What is a typical gas mileage?
8. Estimate the volume of water used per person per day in a typical household. Make separate estimates for bathing, drinking, cooking, washing clothes, and using the toilet.
9. Make an estimate of the amount of food (in pounds) one U.S. resident consumes each year. Include a brief explanation of the steps in this estimate and the result.
10. Make an estimate of the number of automobile tires discarded each year in the United States. Include a brief explanation of the steps in this estimate and the result.

Units

11. Give a common metric unit for mass and energy.
12. Give a common metric unit for volume and power.
13. "Acre-foot" is a unit of which quantity?
14. "Watt-hour" is a unit of which quantity?

Unit Conversion

15. A transect (a linear survey) through a tropical forest measured 13,267 feet in length. How many miles long was this survey?
16. Three and a half barrels of used oil were dumped next to a stream. How many gallons of oil was this?
17. 1.45 km^2 of wetlands were preserved for migrating waterfowl. How large is this area in hectares (ha)?
18. A committee on rail freight transport recommended increasing the mass carried by a trailer from 42 tonnes to 46 tonnes. How big are these masses, in kilograms?
19. A pickup truck was tested using both conventional diesel fuel and **biodiesel** made from vegetable oil. The pickup generated 137 horsepower running on regular diesel and 133 horsepower using biodiesel.[11] Convert these horsepower values to kilowatts.
20. A river otter traveled 60 miles in one season. How many kilometers is 60 miles?
21. The circumference of the Earth is approximately 40,000 km. Convert this distance to miles.
22. According to the Environmental Protection Agency (EPA), the total amount of toxic pollutants released to the U.S. environment in the year 2000 equaled 3.2205 million metric tonnes.[12] How many U.S. tons is this?

Compound Units

23. A small well in a rural village produces 3.5 gallons per minute. What is this discharge in $\frac{m^3}{hour}$?
24. How much mass is 1 m^3 of water, in kilograms? Recall that the density of water is $\frac{1\ \text{gram}}{1\ \text{cm}^3}$.
25. The energy utility PacifiCorp wants to remove its hydroelectric dam on the White Salmon River in Washington State.[13] By 2002, an estimated 2.4 million cubic yards of river sediment (mud, sand, and gravel) had accumulated in the Condit Dam reservoir since the dam was built in 1913. Calculate the average rate of sediment accumulation behind the dam over the 89-year period, both per year and per day.
26. Rain that falls on the Cascade Pole and Lumber Company in Tacoma, Washington, runs into a storm drain and then into the Puyallup River. About 24 million liters of rainwater per year are discharged by this storm drain. This rainfall runoff is contaminated with various chemicals, including arsenic, a toxic heavy metal. The concentration of arsenic in the storm drain water is about 170 micrograms for every liter of stormwater.[14] How many grams and kilograms of arsenic are being dumped into the Puyallup River each year by this storm drain?

Units in Equations

27. The distance water flows down a stream in a certain amount of time is given by

$$\text{distance (meters)} = 1.31\frac{?}{?} \times \text{time (seconds)}$$

What are the units at the question marks?

28. The temperature of the lower atmosphere decreases steadily as the height above the ground increases. The equation is

$$\text{temperature (°C)} = 18°\text{C} - (6.8\frac{?}{?}) \times \text{height (km)}$$

What are the units at the question marks?

Unit Prefixes

29. What is the abbreviation for a million watts of electricity?

30. What is the abbreviation for a thousandth (1/1,000) of a gram of mass?

31. A μm is how many meters in distance?

32. A GPa is how many pascals of pressure?

Scientific Notation and Order of Magnitude

33. In 2001, there were 4,650 species of named mammals compared with 1,025,000 named species of insects.[15] Express each of these abundances in scientific notation. About how many orders of magnitude do insect species exceed mammal species?

34. Brazoria County in Texas produced 12,961,149 tons of toxic waste in 1997, whereas Midland County in Texas produced 6,981 tons of toxics in the same period.[16] Express each of these amounts in scientific notation. By how many orders of magnitude do these counties differ in toxic waste output?

35. The average amount of **PM10** (particulate matter smaller than 10 microns) in Chicago's air was found to be 0.000025 grams of PM10 in a cubic meter of air.[17] The U.S. air quality standard for PM10 is 0.00015 grams per cubic meter of air. Express each of these values in scientific notation using units of grams per cubic meter. By how many orders of magnitude do these two measurements differ?

36. Soil erosion during rainfall events following wildfires in western Montana was measured. Erosion rates during intense, short-duration thunderstorms were as high as 40 tonnes of soil per hectare of land. Erosion rates during low-intensity, long-duration rainfalls averaged about 0.01 tonnes per hectare.[18] Express each of these erosion rates in scientific notation. By how many orders of magnitude do these two rates differ?

Powers of 10 and Logarithms

37. Without using a calculator, what is the log of 10^{-6}? the log of 10^8? the log of 100,000? the log of 0.001?

38. Without using a calculator, what is the log of 10^{-3}? the log of 10^{11}? the log of 10,000,000? the log of 0.000001?

39. Report the answers to the following, rounded to three decimal places. What is the log of 1,731,124? the log of 0.000352? Now express both of the original (unlogged) values in the form 10^x.

40. Report the answers to the following, rounded to three decimal places. What is the log of 71,333? the log of 0.489? Now express both of the original (unlogged) values in the form 10^x.

41. In 2000, lightning-caused wildfires burned 101,013 acres of National Park land and 1,676,414 acres of National Forest land in the United States.[19] Calculate the logarithms of these two acreages. By how many orders of magnitude do these two acreages differ?

42. Fort Lauderdale had a 2003 population of 162,917 residents, whereas Tarpon Springs had a 2003 population of 22,240.[20] Calculate the logarithms of these two populations. By how many orders of magnitude do these two populations differ?

Figure 1-10 A tsunami warning sign along the Oregon coast. Residents should run to high ground following a coastal earthquake or siren warning. *Photo: Langkamp/Hull.*

Logarithmic Scales

43. The December 2004 Indonesian earthquake triggered a giant *tsunami* (translated as "harbor wave"); the earthquake and tsunami killed around 250,000 people (Figure 1-10). The earthquake had a moment of 3.5×10^{29} dyne-centimeters.[21] What is the corresponding magnitude of this earthquake?

44. An earthquake took place off the coast of Mexico in June 2004. The moment for this earthquake is 1.0×10^{24} dyne-centimeters.[22] Is this a large, medium, or small earthquake?

45. A lake at the Wangaloa coal mine in New Zealand has a hydrogen ion concentration of 1.6×10^{-5} moles/liter.[23] What is the pH of this lake water? Is the lake acidic or basic?

46. The hydrogen concentration of Big Moose Lake in the Adirondack Mountains of New York has been monitored since 1992.[24] The average hydrogen concentration during this time period has been about 5.0×10^{-6} moles/liter. What is the pH of the lake water? How does this pH compare with neutral water?

SCIENCE IN DEPTH

Global Warming

With over 6 billion people on Earth, the impacts of humans on the natural environment are beginning to reach planetary proportions. Humans have become a major agent in rearranging the world's landscape, on the same order as landslides, rivers, and wind. About 100 billion metric tonnes of earth materials are moved every year by people.[25] Humans are a potent geologic force!

Humanity has also grown into an important agent of global climate change. Since the beginning of the Industrial Revolution, gases in the atmosphere created by human activities have increased exponentially along with the exponential increase in the human population. Two of these gases, carbon dioxide and methane, are accumulating rapidly in the lower atmosphere. Carbon dioxide in the Northern Hemisphere has increased from about 280 parts per million before the Industrial Revolution to about 380 parts per million today. Methane has increased in the atmosphere from 750 parts per billion to 1,750 parts per billion in this same time period.[26]

Carbon dioxide and methane play an important role in the **energy budget** of the lower atmosphere. Radiation from the Sun passes through the atmosphere, where it is absorbed by dark surfaces such as vegetation and parking lots, and is then reemitted in the form of infrared radiation or heat. Carbon dioxide and methane are both very efficient at absorbing this outgoing infrared radiation. Therefore, any increase in these "greenhouse gases" should produce an increase in the temperature of the lower atmosphere. And indeed, direct measurements around Earth show an average increase in the temperature of the lower atmosphere of 1° Celsius or so over the last 100–150 years.

What are the consequences of human-induced global warming? Global warming has, in some cases, benefited species. For example, pied flycatchers in Germany have had more reproductive success due to an increase in spring temperatures, when these birds lay their eggs and rear their young.[27] Wildflowers in New York State have bloomed earlier, in concert with warming temperatures.[28]

Human-induced global warming has also increased the temperature of the sea surface, with both physical and biological consequences. As sea surface temperatures increase, diseases among coral reefs have become more prevalent, producing bleaching of corals. Coral bleaching has been documented in many of the world's oceans.

In the Arctic, **tree line** has moved northward, with evergreen trees colonizing previously unforested areas, perhaps as a consequence of slightly warmer temperatures. The increased amount of carbon dioxide in the atmosphere may also benefit trees, as plants absorb carbon dioxide from the atmosphere and convert the carbon dioxide to carbohydrates for nourishment. Both extra nourishment of plants and an increase in temperatures may be the result of excess carbon dioxide in the atmosphere.

Appendix: Unit Conversions

DISTANCE

$$1 \text{ centimeter (cm)} = 10 \text{ millimeters (mm)} \approx 0.3937 \text{ inches (in)}$$

$$1 \text{ meter (m)} = 100 \text{ cm} \approx 39.37 \text{ inches} \approx 3.281 \text{ feet} \approx 1.094 \text{ yards}$$

$$1 \text{ kilometer (km)} = 1{,}000 \text{ m} \approx 0.621 \text{ miles}$$

$$1 \text{ mile (mi)} = 5{,}280 \text{ feet} = 1{,}760 \text{ yards} \approx 1.609 \text{ kilometers}$$

AREA

Units of area often come in the form length^2.

$$1 \text{ meter}^2 \approx 10.764 \text{ feet}^2$$

$$1 \text{ acre} = 43{,}560 \text{ ft}^2 \approx 0.405 \text{ hectare}$$

$$1 \text{ hectare (ha)} = 100 \text{ m} \times 100 \text{ m} = 0.1 \text{ km} \times 0.1 \text{ km} = 0.01 \text{ km}^2 \approx 2.47 \text{ acres}$$

$$1 \text{ km}^2 = 1{,}000 \text{ m} \times 1{,}000 \text{ m} = 1 \text{ million m}^2 = 100 \text{ ha} \approx 0.386 \text{ mile}^2$$

$$1 \text{ mile}^2 = 640 \text{ acres} \approx 2.59 \text{ km}^2$$

VOLUME

Units of volume often come in the form length^3.

$$1 \text{ cm}^3 = 1 \text{ cc (cubic cm)} = 1 \text{ milliliter} \left(\text{mL or } \frac{1}{1{,}000} \text{ liter}\right) \approx 0.0338 \text{ fluid ounces}$$

$$1 \text{ liter} = 1{,}000 \text{ mL} = 1{,}000 \text{ cm}^3 \approx 1.057 \text{ quarts (liquid)}$$

$$1 \text{ gallon (gal)} = 4 \text{ quarts} = 8 \text{ pints} = 16 \text{ cups} \approx 3.785 \text{ liters} \approx 0.1337 \text{ ft}^3$$

$$1 \text{ m}^3 = 1{,}000 \text{ liters} \approx 264.172 \text{ gallons} \approx 35.315 \text{ ft}^3$$

$$1 \text{ barrel of oil} = 42 \text{ gallons of oil} \approx 158.987 \text{ liters of oil} \approx 0.158987 \text{ m}^3 \text{ of oil}$$

$$1 \text{ bushel of grain} \approx 0.03524 \text{ m}^3 \approx 1.244 \text{ ft}^3$$

$$1 \text{ acre-foot of water} = 1 \text{ acre} \times 1 \text{ foot} = 43{,}560 \text{ ft}^3 \approx 325{,}851 \text{ gallons}$$

DISCHARGE

Discharge is volume divided by time and is a common measure of streamflow or irrigation rate.

$$1 \frac{\text{ft}^3}{\text{sec}} \text{ (cubic feet per sec or cfs)} \approx 7.481 \frac{\text{gal}}{\text{sec}}$$

$$1 \frac{\text{m}^3}{\text{sec}} \approx 35.315 \frac{\text{ft}^3}{\text{sec}}$$

$$\frac{1 \text{ acre-foot}}{1 \text{ day}} \approx 226.29 \frac{\text{gallons}}{\text{minute}} \text{ (gallons per minute or gpm)}$$

MASS AND WEIGHT (ON EARTH)

Mass (the amount of matter) is *not* the same as weight (the force on an object due to gravity). Units of mass include the gram or kilogram (metric) while units of weight include the Newton (metric) and the pound (English). We can, however, equate the kilogram (mass) with the pound (weight), even though these measures belong to different families of units, as long as we stay on Earth!

$$1 \text{ dry ounce (oz)} \approx 28.35 \text{ grams (gm or g)}$$
$$1 \text{ pound (lb)} = 16 \text{ dry ounces (oz)} \approx 0.454 \text{ kilograms (kg)}$$
$$1 \text{ kilogram (kg)} = 1{,}000 \text{ grams} = 1 \text{ liter of water} \approx 2.205 \text{ pounds (on Earth)}$$
$$1{,}000 \text{ kilograms} = 1 \text{ tonne (metric ton)} \approx 2{,}204.6 \text{ pounds}$$
$$2{,}000 \text{ pounds} = 1 \text{ ton (U.S. ton or "short" ton)} \approx 0.9072 \text{ tonnes}$$

DENSITY

Density (also known as specific gravity) is mass divided by volume or mass per volume. Lead and gold are dense, whereas chicken feathers are "light" (low density). The density of pure water at room temperature is 1 gram per cubic centimeter (1 gram per milliliter).

$$\frac{1 \text{ g}}{1 \text{ cm}^3} = \frac{1{,}000 \text{ kg}}{1 \text{ m}^3}$$

The term *density* can also refer to concentration, such as parts per million.

FORCE

Force is mass times acceleration. When the acceleration is due to gravity, gravitational force is often called "weight" (see previous section).

$$1 \text{ dyne} = 1 \text{ g} \times 1 \frac{\text{cm}}{\text{sec}^2}$$
$$1 \text{ newton (Nt or N)} = 1 \text{ kg} \times 1 \frac{\text{m}}{\text{sec}^2} = 10^5 \text{ dynes}$$

PRESSURE

Pressure is force divided by the area on which the force is acting. One atmosphere is the typical pressure on the Earth's surface caused by the weight of gases in the atmosphere. If you dive to the bottom of a 30-foot-deep swimming pool, you can feel an extra 1 atmosphere of pressure due to the weight of the water.

$$10 \frac{\text{dyne}}{\text{cm}^2} = 1 \frac{\text{newton}}{\text{m}^2} = 1 \text{ pascal (Pa)}$$
$$\frac{1 \text{ pound}}{1 \text{ in}^2} \text{ (pounds per square inch or psi)} \approx 0.068 \text{ atmospheres}$$
$$1 \text{ bar} = 10^5 \text{ pascals (Pa)} \approx 0.987 \text{ atmospheres}$$

WORK, MOMENT, ENERGY, AND HEAT

These four physical concepts have the same units. Work or moment is a force multiplied by the distance through which the force acts. If you have used a tire wrench to take off lug nuts, you are familiar with both moment and work! Energy is the capacity or ability to do work and comes in many forms, such as kinetic energy or potential energy. Heat is the amount of energy required to change the temperature of an object.

$$1 \text{ erg} = 1 \text{ dyne} \times 1 \text{ cm}$$
$$1 \text{ joule (J)} = 1 \text{ newton} \times 1 \text{ meter} = 1 \text{ N-m} = 10^7 \text{ ergs}$$
$$1 \text{ calorie (cal, with small c)} \approx 4.1868 \text{ joules}$$
$$1{,}000 \text{ calories} = 1 \text{ kilocalorie (kcal)} = 1 \text{ Calorie (capital C; "the food" calorie)}$$
$$1 \text{ British thermal unit (BTU)} \approx 1{,}055.056 \text{ joules} = 1.055 \text{ kJ}$$

Power is energy per time (see the next section). Therefore,

$$\text{power} = \frac{\text{energy}}{\text{time}}$$
$$\text{power} \times \text{time} = \left(\frac{\text{energy}}{\cancel{\text{time}}}\right) \times \cancel{\text{time}} = \text{energy}$$

Somewhat perversely, the amount of energy produced or consumed is often expressed in units of "power-time." Watt is a common unit of power (see the next section); therefore,

$$1 \text{ joule} = 1 \text{ watt} \times 1 \text{ sec} = 1 \text{ watt-sec}$$
$$3{,}600 \text{ joules} = 1 \text{ watt} \times 1 \text{ hour} = 1 \text{ watt-hour} = 1 \text{ Wh}$$
$$1{,}000 \text{ watt-hours} = 1 \text{ kilowatt-hour} = 1 \text{ kWh}$$

POWER

Power is work or energy divided by time and is the rate at which energy is produced or consumed. Power is work per time or energy per time.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ sec}} \approx 3.413 \frac{\text{BTU}}{\text{hour}}$$
$$1 \text{ horsepower} \approx 0.7457 \text{ kilowatts (kW)}$$

TEMPERATURE

There are three common scales of temperature; Celsius (sometimes mislabeled as "centigrade"), Fahrenheit, and Kelvin. Plain water freezes at 0°C (32°F or 273.15 K) and boils at 100°C (212°C and 373.15 K). No degree symbol is used for Kelvin!

$$\text{Celsius} = \frac{5}{9}(\text{Fahrenheit} - 32°)$$
$$\text{Fahrenheit} = \frac{9}{5}\text{Celsius} + 32°$$
$$\text{Kelvin} \approx \text{Celsius} + 273.15$$

ENERGY CONTENT

How much energy is contained in common fuels if they're burned efficiently? Note that these relationships are not "unit equivalents"; pounds and gallons do not belong to the same category as BTUs. Some conversions may vary.

$$1 \text{ cubic foot of natural gas} = 1{,}026 \text{ BTU}$$
$$1 \text{ pound of coal} = 10{,}340 \text{ BTU}$$
$$1 \text{ gallon of propane} = 91{,}000 \text{ BTU}$$
$$1 \text{ gal gasoline} = 0.898 \text{ gal crude oil} = 0.8921 \text{ gal heating oil}$$
$$1 \text{ gal gasoline} = 0.828 \text{ gal fuel oil} = 124{,}000 \text{ BTU}$$

RADIATION AND DOSAGE

The Curie and the Becquerel are units of radiation, whereas the rad, Gray, rem, and Sievert are units of exposure or dose.

$$1 \text{ becquerel (Bq)} = \frac{1 \text{ radioactive decay event}}{1 \text{ second}} = \frac{1}{\text{second}}$$

$$1 \text{ curie(Ci)} = \frac{37 \times 10^{9} \text{ decay events}}{1 \text{ second}} = 37 \text{ GBq}$$

$$100 \text{ radiation absorbed doses} = 100 \text{ rad} = 1 \text{ gray (Gy)} = 1 \frac{\text{joule}}{\text{kilogram}}$$

$$100 \text{ roentgen equivalent mammal} = 100 \text{ rem} = 1 \text{ sievert (Sv)}$$

MOLE

Some familiar terms that tell us "how many" include a dozen (12) doughnuts or a ream (500 sheets) of paper. Chemists have their own term for "how many" atoms or molecules. This term is the **mole** and is a very large number, approximately 6.022×10^{23}, almost a trillion trillion. If you have one mole of carbon atoms, or a mole of water molecules, you have 6.022×10^{23} carbon atoms or water molecules. Because individual atoms and molecules are so small, a mole of carbon or water isn't a lot of mass; a mole of carbon is 12 grams (less than half an ounce), and a mole of water is about 18 grams.

Moles show up occasionally when discussing concentration (see Chapter 1). For example, the amount of nitrate in seawater can be expressed as "microMoles of nitrate per liter of water" or μM/L. A liter of water contains about 56 moles of water. Therefore, one microMole in one liter equals

$$\frac{1 \text{ microMole}}{\text{liter}} = \frac{10^{-6} \text{ Moles}}{56 \text{ Moles}} = \frac{10^{-6} \cancel{\text{Moles}}}{56 \cancel{\text{Moles}}} = 1.79 \times 10^{-8}$$

$$1.79 \times 10^{-8} = \frac{1.79 \times 10^{-8} \times 1 \text{ billion}}{1 \text{ billion}} = \frac{17.9}{1 \text{ billion}} = 17.9 \text{ ppb}$$

One microMole per liter equals about 18 parts per billion.

PREFIXES

Prefix	Symbol	Value	Exponential Equivalent	Example Using Prefix
peta	P	1 quadrillion	10^{15}	petagram (Pg)
tera	T	1 trillion	10^{12}	terabyte (Tbyte)
giga	G	1 billion	10^{9}	gigawatt (GW)
mega	M	1 million	10^{6}	mega-annum (Ma)
kilo	k	1 thousand	10^{3}	kilopascal (kPa)
milli	m	1 thousandth	10^{-3}	milliliter (mL)
micro	μ	1 millionth	10^{-6}	micrometer (μm)
nano	n	1 billionth	10^{-9}	nanogram (ng)
pico	p	1 trillionth	10^{-12}	picosecond (psec)

Answers to Odd-Numbered Exercises

Chapter 1 Exercises

1. Clam shell. *Answers may vary.* 5.5 cm long and 4.1 cm wide. Choice of axes may vary.
3. Reservoir evaporation. 0.07 inches per day less than the measured value. Accuracy seems moderate.
5. MTBE in groundwater. Precision seems very high, within one part out of a billion parts.
7. Gallons of gasoline. 10,000 miles per year at 20 miles per gallon equals 500 gallons of gasoline.
9. Food consumption. 1 pound of food per meal three times a day equals 1,095 pounds per year.
11. Kilogram and joule (among others)
13. Volume
15. Forest transect

$$13{,}267 \cancel{\text{feet}} \times \frac{1 \text{ mile}}{5{,}280 \cancel{\text{feet}}} \approx 2.51 \text{ miles}$$

17. Wetlands

$$1.45 \cancel{\text{km}^2} \times \frac{100 \text{ ha}}{1 \cancel{\text{km}^2}} = 145 \text{ ha}$$

19. Pickup power

$$137 \text{ hor}\cancel{\text{sepower}} \times \frac{0.7457 \text{ kilowatts}}{1 \text{ hor}\cancel{\text{sepower}}} \approx 102.16 \text{ kW}$$

compared with 99.18 kW.

21. Earth's circumference

$$40{,}000 \cancel{\text{km}} \times \frac{1 \text{ mile}}{1.609 \cancel{\text{km}}} \approx 24{,}860 \text{ miles}$$

23. Discharge

$$\frac{3.5 \text{ gallons}}{\cancel{\text{minute}}} \times \frac{60 \cancel{\text{minutes}}}{1 \text{ hour}} = \frac{210 \text{ gallons}}{1 \text{ hour}}$$

$$\frac{210 \cancel{\text{gallons}}}{1 \text{ hour}} \times \frac{1 \text{ m}^3}{264.172 \cancel{\text{gallons}}} = 0.795 \frac{\text{m}^3}{\text{hour}}$$

25. River sediment

$$\frac{2.4 \text{ million cubic yards}}{89 \text{ years}} = \frac{26{,}966 \text{ yd}^3}{1 \text{ year}}$$

$$\frac{26{,}966 \text{ yd}^3}{1 \text{ year}} \times \frac{1 \text{ year}}{365 \text{ days}} = 73.9 \frac{\text{yd}^3}{\text{day}}$$

27. Meters per second
29. Megawatt (MW)
31. One-millionth (10^{-6}) of a meter
33. Mammals versus insects. 4.65×10^{3} mammals and 1.025×10^{6} insects. 2–3 orders of magnitude.
35. PM10. $0.000025 \frac{\text{g}}{\text{m}^3}$ is $2.5 \times 10^{-5} \frac{\text{g}}{\text{m}^3}$, whereas $0.00015 \frac{\text{g}}{\text{m}^3}$ is $1.5 \times 10^{-4} \frac{\text{g}}{\text{m}^3}$. One order of magnitude.
37. $-6, 8, 5$, and -3
39. 6.238 and -3.453
41. Wildfire acreages. 5.004 and 6.224. 1 order of magnitude.
43. Indonesian earthquake magnitude

$$\log\left((3.5 \times 10^{29} \text{ dyne} \times \text{cm})^{\frac{2}{3}} / 10^{10.7}\right) \approx 9.0$$

45. Mine drainage
    $\text{pH} = -\log(1.6 \times 10^{-5} \text{ moles/liter}) \approx 4.8$
    The lake water is rather acidic.

Chapter 2 Exercises

1. Red List

$$\frac{15{,}042 \text{ species}}{10{,}731 \text{ species}} \approx 1.40$$

3. Land areas

$$\text{California's area} = \frac{4 \text{ million km}^2}{10} = 400{,}000 \text{ km}^2$$

5. California power plants
   a.

Plant Name	Normalized (tons/TBTU)
Santa Clara	0.058
SCA	0.521
Scattergood	0.054
South Bay	0.059
SPA	0.059
Walnut	0.058

   b. With one exception (SCA), all the power plants have very similar ratios.
   c. SCA
7. San Francisco automobiles

Year	Normalized (cars/person)
1930	0.293
1940	0.353
1950	0.375
1960	0.445
1970	0.541
1980	0.634
1990	0.656
2000	0.698